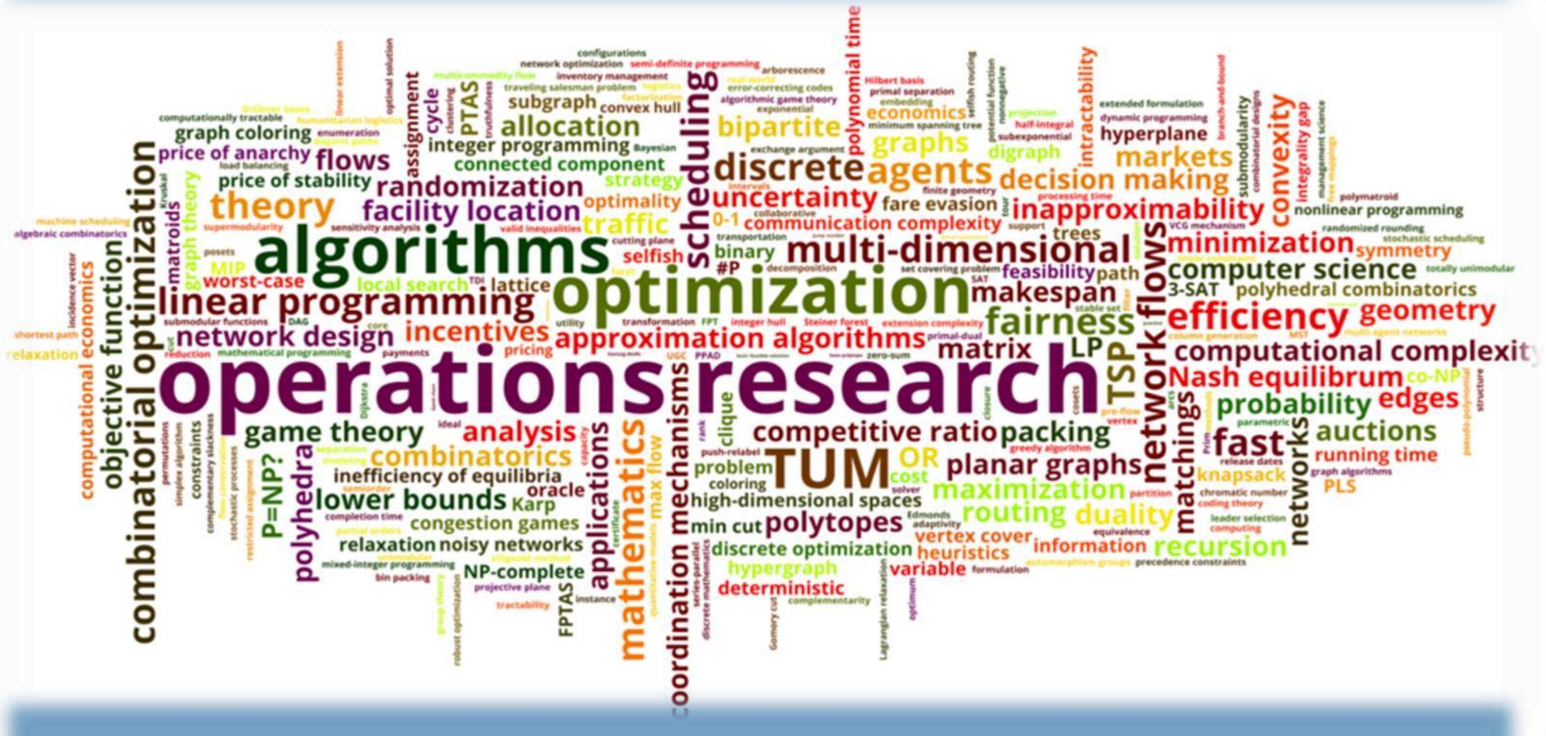
College of Defence Management



A Primer to

Operations Research

Pre Course Training Handout

PRE-COURSE TRAINING HANDOUT : OPERATIONS RESEARCH

PRIMER TO OPERATIONS RESEARCH (OR)

"Nothing is more difficult, and therefore more precious, than to be able to decide."

- Napoleon Bonaparte

General

1. Operations Research effectively addresses tactical, operational and Strategic problems and provides leadership in global, military and business environment. Decision makers cannot afford to take decisions based on personal experiences/ intuitions in such scenarios because the consequences of failures in such situations may prove very costly. Hence, there is a need for structural analysis using operations research/ quantitative techniques to arrive at a holistic solution to any managerial problem. This can be done by critically analyzing the levels of interaction between the applications process of operations research and various systems of an organization.

2. OR was first used during World War II to assist high-level decision makers with analyses that could be used to support planning for strategic ground, air and maritime operations. A group of scientists were attached to the military staffs in the UK and USA to help in finding solutions to tactical problems by collecting and scientifically analysing military data. Since then, OR techniques have been used to solve many sophisticated and complex defence-related problems, not only limited to combat operations but also encompassing logistics, manpower planning, equipment procurement, training, infrastructure and many other areas. The global military community has fully embraced OR as a critical tool for ensuring efficiency and readiness.

3. On successful completion of the course, participants will be enabled to effectively utilize various OR tools and techniques to solve complex tactical, operational and strategic issues in their respective services.

Definition

4. An apt definition of OR in the domain of Armed Forces is: -

"Employment of scientific methods of analysis to provide commanders with quantitative basis for making decisions regarding operations under their control".

Historical Perspective

5. During the Second World War, the British and American forces realised that military knowledge by itself was not adequate to analyse military systems for finding the best or optimum solutions to their tactical problems. Mathematicians and scientists from various disciplines were attached to the military staff in these two countries. The usefulness of their work was acknowledged and towards the end of the Second World War this new discipline came to be known as "Operational Research" or "Operational Analysis".

Techniques of OR

6. Decision situations may be classified into three categories i.e. Deterministic, Risk and Uncertainty. To cope with the varying degrees of uncertainty in problem solving, OR practitioners have progressively used and developed various categories of techniques. Each of these categories have a number of techniques or models to solve a specific type of problem. Some of the more commonly used models that would be covered during the course are given in the succeeding paragraphs.

7. **Linear Programming (LP)**. This is the most commonly used technique and it deals with the allocation of scarce resources in an optimal manner so as to maximise effectiveness or minimise cost e.g. best mix of weapons, aircrafts, manpower etc., to achieve a specified objective. A LP model can handle any number of variables and their constraints simultaneously. Special models, as variations of LP, have been developed to solve certain specific types of problems. These are: -

(a) **Transportation Model**. Problems dealing with matching of sources of supply to destinations on basis of cost, distance or time e.g., movement of stores from depots to forward areas; selection of firms for supply of stores based on availability or cost. Move of stores is allowed only between sources and destinations.

(b) **Transshipment Model**. It is a variation of Transportation Model in which stores are allowed to be moved within the sources as well as the destinations.

(c) **Assignment Model**. To assign a number of 'origins' to a number of 'destinations' at a minimum total cost e.g. assigning men/machines/units to some number of jobs/tasks. Cost may be in terms of time, distance, satisfaction levels or payoffs etc.

8. **Networks.** These models are used to connect all nodes of a network with the shortest dist possible, connect two points with the shortest route or calculate max flow feasible between a source and the destination.

9. **Analytical Hierarchy Process (AHP)**. AHP allows to carry out a fair and objective selection of personnel or equipment/weapons against laid down

criteria/desired attributes when the choices are closely competing i.e., they all are equally good.

10. **Queuing Model**. This model helps to determine the amount of servicing facilities to be provided to reduce delays to acceptable levels, considering random pattern of arrivals and servicing time; and optimal utilisation of resources, e.g., number of doctors/beds to be provided in a hospital, operators in a signal centre, berths in a harbour, etc.

11. **Simulation Model**. It deals with quantifying risk/probability of occurrence of the phenomenon based on probabilistic estimates. It can simulate behaviour pattern of queues, weapons/weapon systems, the outcome of engagements in a battle, the breakdown of equipment, repair facilities etc. It is like an experimental laboratory available to an OR practitioner and is one of the most powerful techniques for analysing existing and future systems.

12. **Decision Matrix (Expected Value Models)**. Simultaneous assessment of feasible and alternate courses of action in the face of a number of different environments (states of nature) with assessed probabilities of occurrence e.g., selection of own best course of action from various alternatives considering different courses of action open to the enemy.

13. **Decision Tree**. Basic approach is similar to the decision matrix. Whereas, decision matrix deals with single point decision problems, the decision tree analyses sequential problems e.g., multiphase attack/advance operation.

14. The list of techniques discussed above is not exhaustive and there are a number of other techniques as well.

Method of Solving Problems

15. All OR problems can be solved using :-

(a) **Manual Calculations**. Uses iterative processes like Simplex Method. It is a long drawn complex process prone to inadvertent mistakes due to repetitive calculations and iterations.

(b) **Licensed Softwares**. Softwares like POM/QM, Expert Choice etc make the job much easier, however, they are expensive and the free versions have limited functionality.

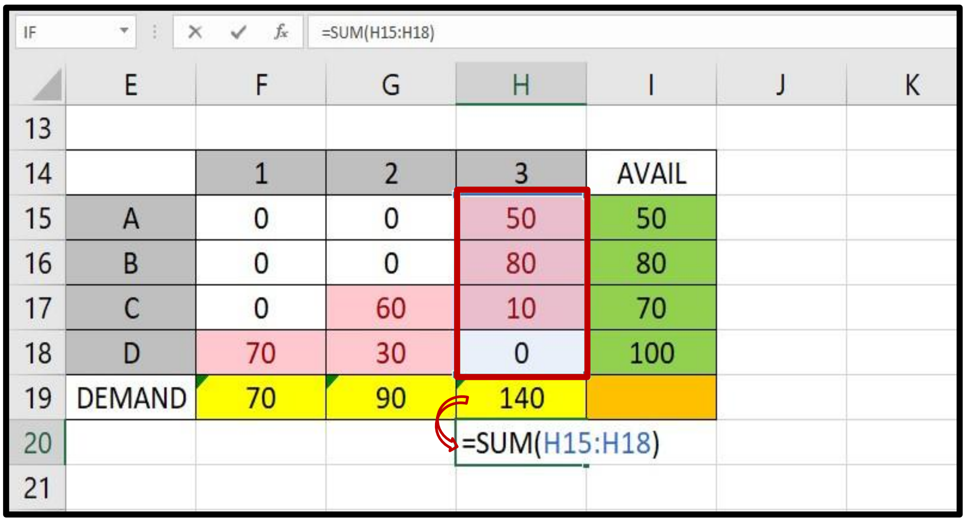
(c) **MS Excel**. The versatility and ease of availability of MS Excel makes it a very convenient choice. Hence, it forms the backbone of all calculations in OR during the course and basic working knowledge of MS Excel is inescapable.

MS Excel

16. A handout on **'Introduction to MS Excel'** has already been hosted on the CDM website for the participants which covers the basics of Excel worksheet along with the essential functions and formulae. The functions which are specific to OR and used extensively in solving the OR Problems are explained in the succeeding paragraphs.

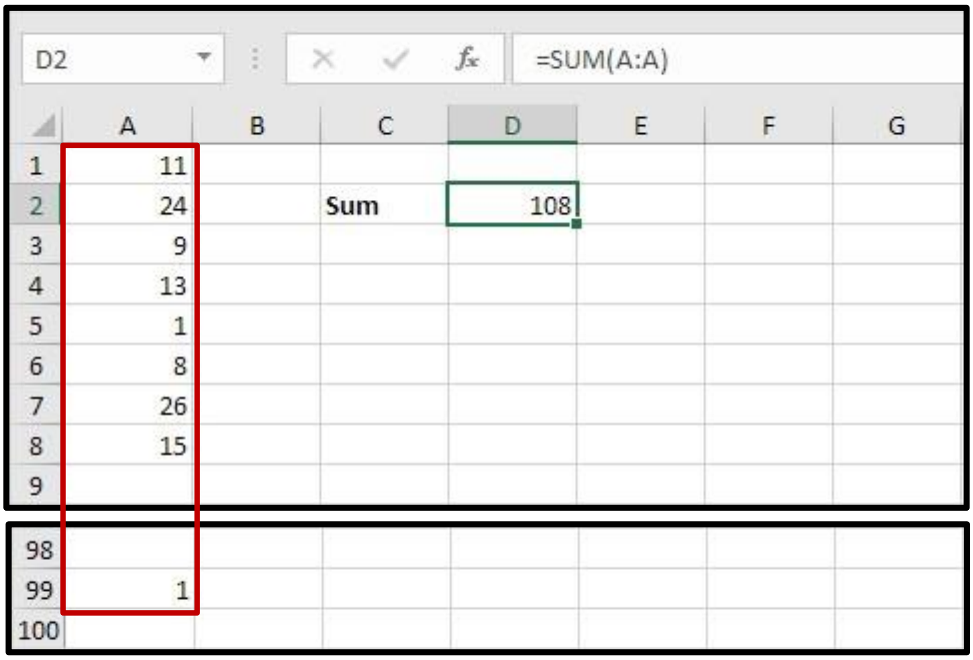
17. **SUM**. One of the most used functions is the SUM function. This function allows you to add the values in a range of cells, an entire column, an entire row or non-contiguous cells. The function is written as: =SUM(range or ranges to add). You can type the function and then use the pointing technique to fill in the arguments. You can add individual values, cell references or ranges or a mix of all three.

(a) **Sum Range**. Most of the time, you'll use the SUM function in Excel to sum a range of cells. For example; =SUM(H15:H18). It adds the values in cells H15:H18 and give the soln as 140 . SUM(H15:H18, G15:G18) adds the values in cells H15:H18 as well as cells G15:G18 and will give the soln as 230.


IF | =SUM(H15:H18)

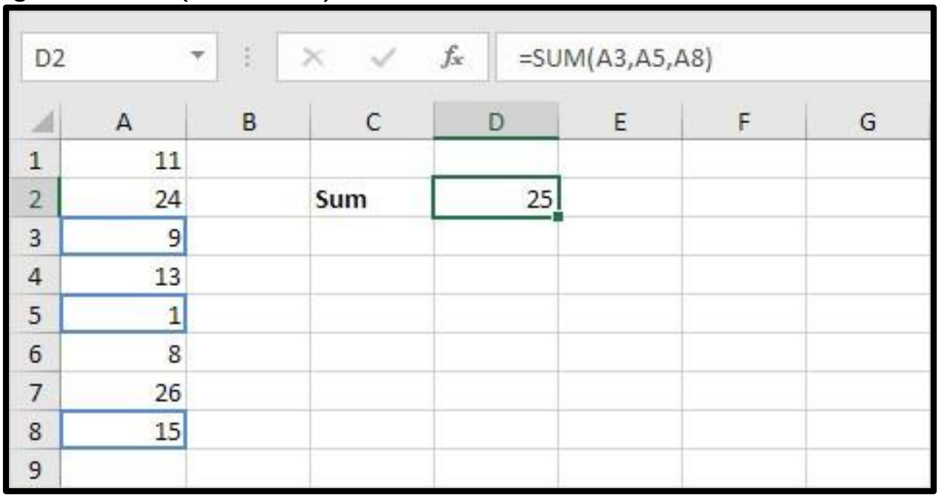
	E	F	G	H	I	J	K
13							
14		1	2	3	AVAIL		
15	A	0	0	50	50		
16	B	0	0	80	80		
17	C	0	60	10	70		
18	D	70	30	0	100		
19	DEMAND	70	90	140			
20				=SUM(H15:H18)			
21							

(b) **Entire Column / Row**. You can also use the SUM function in Excel to sum an entire A column using =SUM(A:A) and for entire 5th row using =SUM(5:5).



	A	B	C	D	E	F	G
1	11						
2	24		Sum	108			
3	9						
4	13						
5	1						
6	8						
7	26						
8	15						
9							
98							
99	1						
100							

(c) **<u>Sum Non-contiguous Cells</u>.** You can also use the SUM function in Excel to sum non-contiguous cells. Non-contiguous means not next to each other eg. D2=SUM(A3,A5,A8).

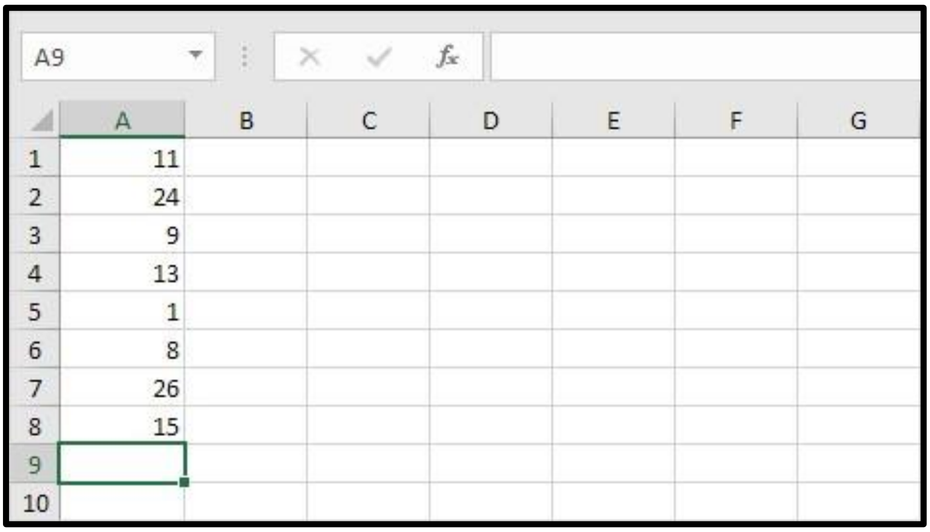


	A	B	C	D	E	F	G
1	11						
2	24		Sum	25			
3	9						
4	13						
5	1						
6	8						
7	26						
8	15						
9							

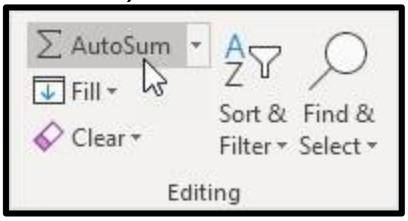
*Note: =A3+A5+A8 produces the exact same result!

(d) **<u>AutoSum</u>**. Use AutoSum or press ALT + = to quickly sum a column or row of numbers.

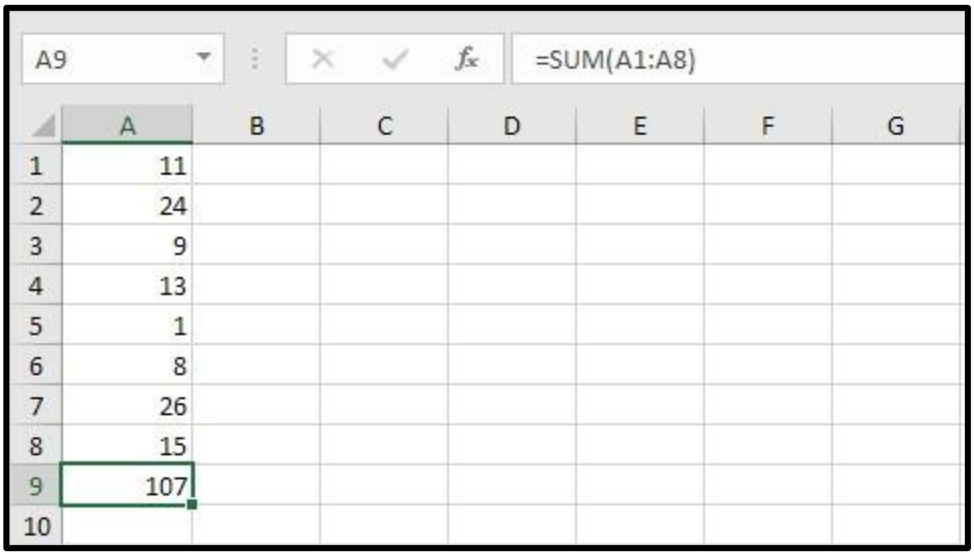
(i) First, select the cell below the column of numbers (or next to the row of numbers) you want to sum.



(ii) On the Home tab, in the Editing group, click AutoSum (or press ATL + =).



(iii) Press Enter.



18. **<u>SUMPRODUCT</u>.** This function is used to calculate the sum of the products of corresponding numbers in one or more ranges or arrays. Syntax =SUMPRODUCT (array1, array2) where array1 and array2 are the first and the second array whose vales are to be multiplied and added.

(a) **<u>Ranges</u>**. The value of 70 in cell F6 is the SUMPRODUCT of range C5:E5 and range C6:E6 i.e. (20*2)+(10*3)+(0*2.5) = 70

IF | fx =SUMPRODUCT(C5:E5,C6:E6)

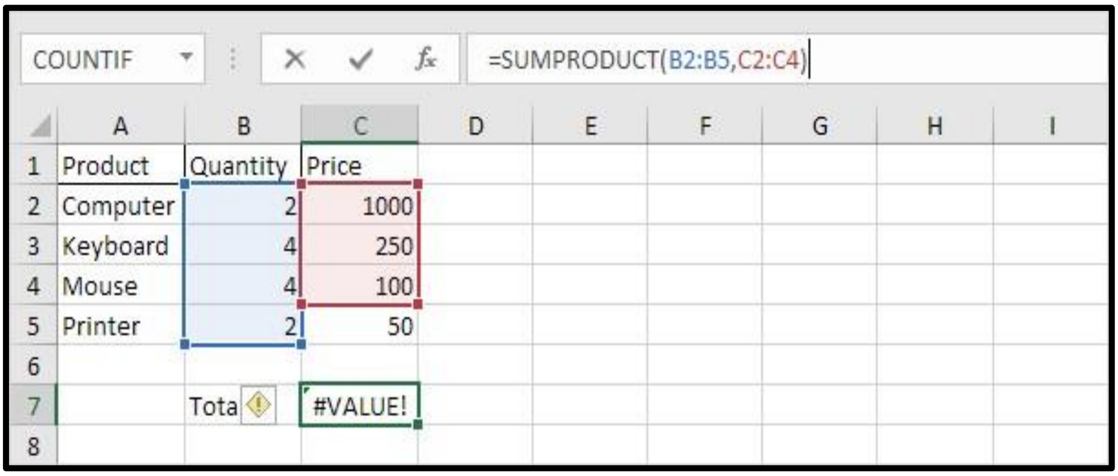
	A	B	C	D	E	F	G	H	I
3									
4		DV	A	B	C				
5			20	10	0				
6		OBJ FN	2	3	2.5	70		=SUMPRODUCT(C5:E5,C6:E6)	
7		FUEL	10	20	16	400			
8		CREW	1	1	1	30			
9		AVBL A	1			20			
10		AVBL B		1		10			
11		AVBL C			1	0			

(b) **<u>Arrays</u>**. Value of 180 in the cell F12 is the SUMPRODUCT of array F6:H9 and array F15:H18 i.e., (2*0+7*0+4*50+3*0+3*0+1*80+5*0+4*60+7*10+1*70+6*30+2*0 = 840)

G12 | fx =SUMPRODUCT(F6:H9,F15:H18)

	E	F	G	H	I	J	K
4			CORPS				
5		1	2	3	AVAIL		
6	A	2	7	4	50		
7	B	3	3	1	80		
8	C	5	4	7	70		
9	D	1	6	2	100		
10	DEMAND	70	90	180	300		
11							
12	Z Min	840	=SUMPRODUCT(F6:H9,F15:H18)				
13							
14		1	2	3	AVAIL		
15	A	0	0	50	50		
16	B	0	0	80	80		
17	C	0	60	10	70		
18	D	70	30	0	100		
19	DEMAND	70	90	140			
20							

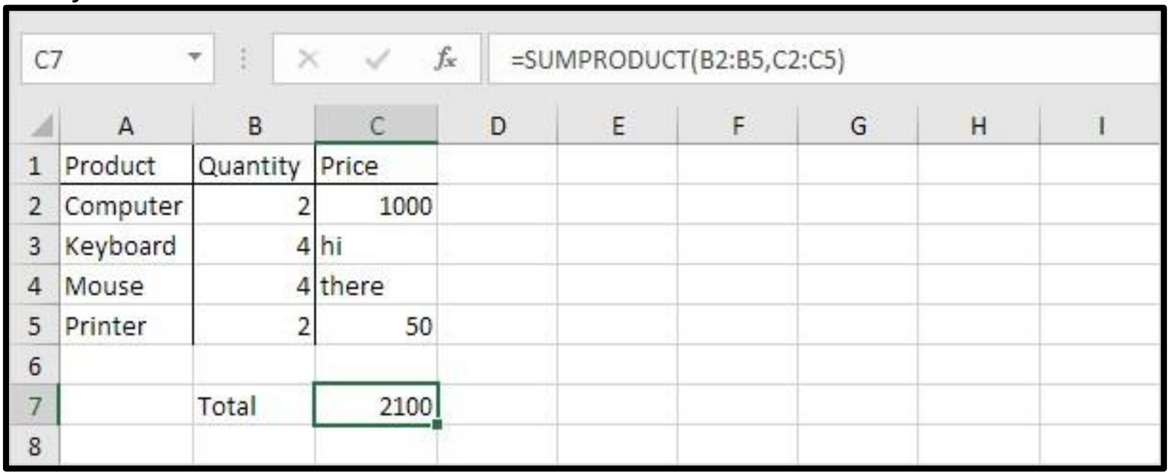
(c) The ranges must have the same dimensions or Excel will display the #VALUE! Error



COUNTIF | =SUMPRODUCT(B2:B5,C2:C4)

	A	B	C	D	E	F	G	H	I
1	Product	Quantity	Price						
2	Computer	2	1000						
3	Keyboard	4	250						
4	Mouse	4	100						
5	Printer	2	50						
6									
7		Tota	#VALUE!						
8									

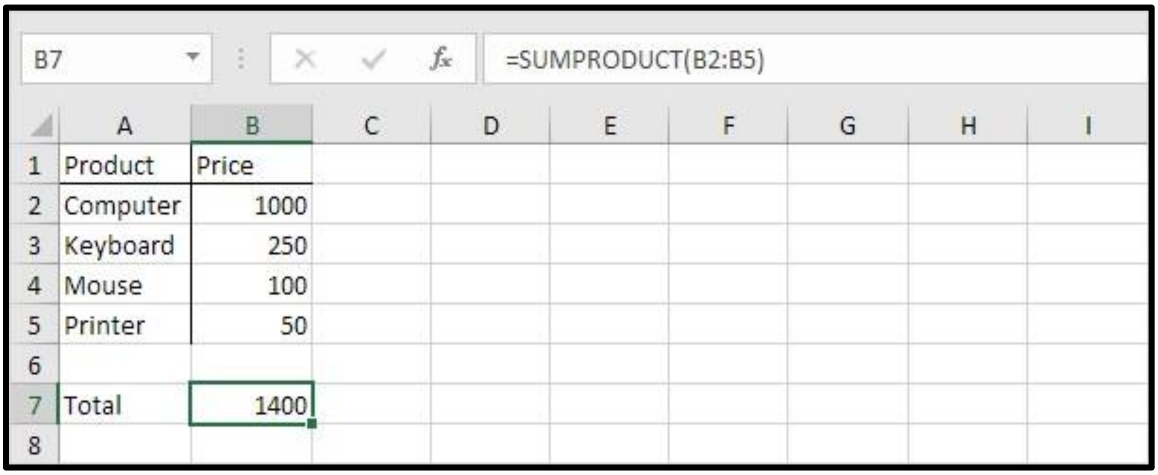
(d) The SUMPRODUCT function treats any entries that are not numeric as if they were zeros.



C7 | =SUMPRODUCT(B2:B5,C2:C5)

	A	B	C	D	E	F	G	H	I
1	Product	Quantity	Price						
2	Computer	2	1000						
3	Keyboard	4	hi						
4	Mouse	4	there						
5	Printer	2	50						
6									
7		Total	2100						
8									

(e) If you supply a single range, the SUMPRODUCT function produces the exact same result as the SUM function.



B7 | =SUMPRODUCT(B2:B5)

	A	B	C	D	E	F	G	H	I
1	Product	Price							
2	Computer	1000							
3	Keyboard	250							
4	Mouse	100							
5	Printer	50							
6									
7	Total	1400							
8									

19. **MMULT**. The MMULT function returns the matrix product of two arrays. The result is an array with the same number of rows as array 1 and the same number of

columns as array 2. For example, you can multiply a 3 x 3 array by a 3 x 1 array to return a 3 x 1 array result. The formula must be entered as an array formula by first selecting the output range, entering the formula in the top-left-cell of the output range, and then pressing CTRL+SHIFT+ENTER to confirm it. Excel inserts curly brackets at the beginning and end of the formula.

(a) Syntax =MMULT (array1, array2) where array1 and array2 are the first and the second array to multiply.

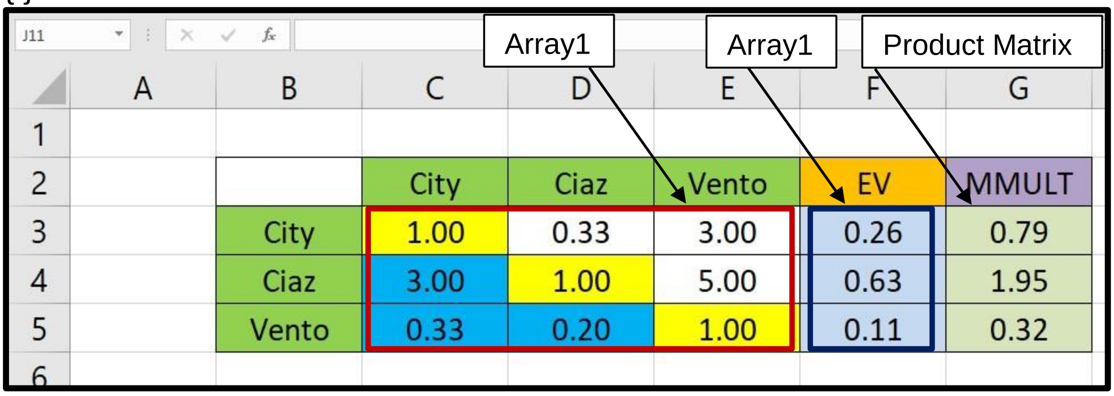
(b) Arrays must contain only numbers.

(c) Columns in array1 must equal the rows in array2.

(d) Array1 and Array2 can be provided as cell ranges, array constants or references.

(e) MMULT returns the #VALUE! error if any cells in array1 and array2 are not numbers, or if array1 columns do not equal array2 rows.

(f) In the example below; we are multiplying a 3x3 matrix (array1) with a 3x1 matrix (array2). The resultant matrix will be a 3x1 matrix. For entering the MMULT function, we selected array G3:G5 and entered the function call as ***=MMULT(C3:E5, F3:F5)*** and ***pressed*** **CTRL+SHIFT+ENTER** to get the answer**.** Please note that the formula now will be surrounded by curly braces { }.



	A	B	C	D	E	F	G
1							
2			City	Ciaz	Vento	EV	MMULT
3		City	1.00	0.33	3.00	0.26	0.79
4		Ciaz	3.00	1.00	5.00	0.63	1.95
5		Vento	0.33	0.20	1.00	0.11	0.32
6							

20. **<u>VLOOKUP</u>**. VLOOKUP returns a cell value from the adjacent columns of a reference table corresponding to a lookup value in the first column. Syntax is =VLOOKUP(lookup value, table array, col index number, range lookup). In its simplest form, the VLOOKUP function says:

=VLOOKUP(What you want to look up, where you want to look for it, the column number in the range containing the value to return, return an Approximate or Exact match – indicated as 1/TRUE, or 0/FALSE). Following info is reqd to build the syntax:-

(a) The value you want to look up, also called the lookup value.

(b) The range where the lookup value is located. Remember that the lookup value should always be in the first column in the range for VLOOKUP to work correctly. For example, if your lookup value is in cell C2 then your range should start with C.

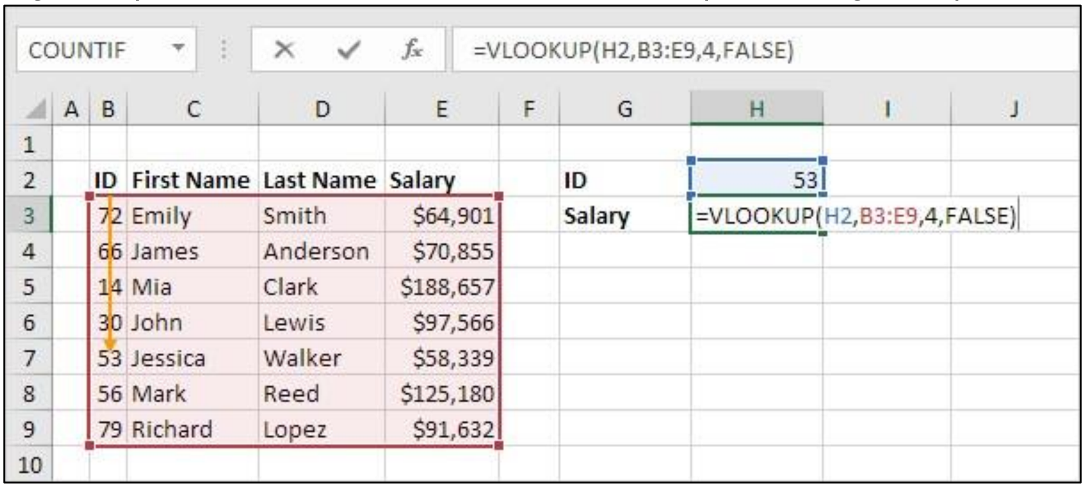
(c) The column number in the range that contains the return value. For example, if you specify B2:D11 as the range, you should count B as the first column, C as the second, and so on.

(d) Optionally, you can specify TRUE if you want an approximate match or FALSE if you want an exact match of the return value. If you don't specify anything, the default value will always be TRUE or approximate match. Now put all of the above together as follows:

=**VLOOKUP**(lookup value, range containing the lookup value, the column number in the range containing the return value, Approximate match (TRUE) or Exact match (FALSE)).

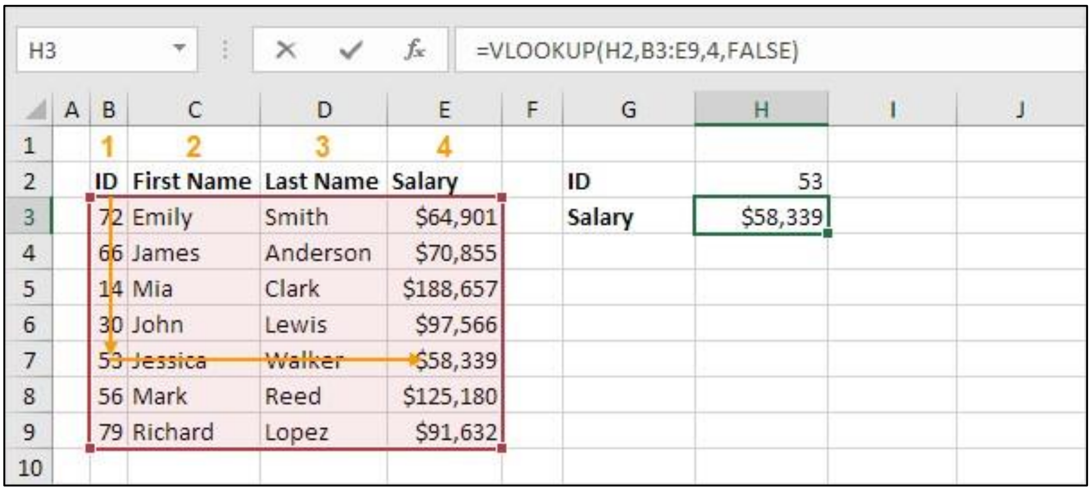
21. Most of the time you are looking for an exact match when you use the VLOOKUP function in Excel. Some of the arguments of the VLOOKUP function are as below:-

(a) **Exact Match**. The VLOOKUP function below looks up the value 53 (first argument) in the leftmost column of the red table (second argument).


COUNTIF =VLOOKUP(H2,B3:E9,4,FALSE)

	A	B	C	D	E	F	G	H	I	J
1										
2		ID	First Name	Last Name	Salary		ID	53		
3		72	Emily	Smith	$64,901		Salary	=VLOOKUP(H2,B3:E9,4,FALSE)		
4		66	James	Anderson	$70,855					
5		14	Mia	Clark	$188,657					
6		30	John	Lewis	$97,566					
7		53	Jessica	Walker	$58,339					
8		56	Mark	Reed	$125,180					
9		79	Richard	Lopez	$91,632					
10										

(b) The value 4 (third argument) tells the VLOOKUP function to return the value in the same row from the fourth column of the red table.

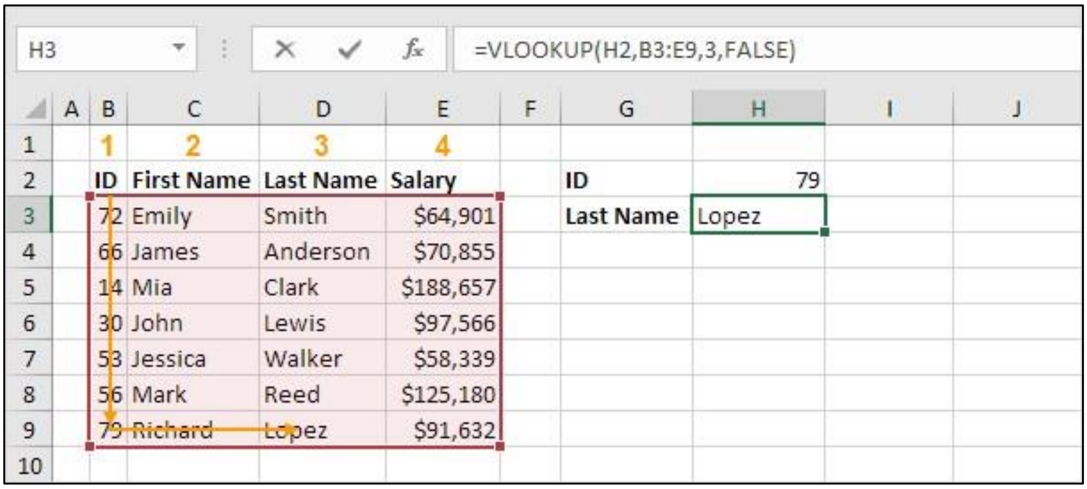


H3 =VLOOKUP(H2,B3:E9,4,FALSE)

	A	B	C	D	E	F	G	H	I	J
1		1	2	3	4					
2		ID	First Name	Last Name	Salary		ID	53		
3		72	Emily	Smith	$64,901		Salary	$58,339		
4		66	James	Anderson	$70,855					
5		14	Mia	Clark	$188,657					
6		30	John	Lewis	$97,566					
7		53	Jessica	Walker	$58,339					
8		56	Mark	Reed	$125,180					
9		79	Richard	Lopez	$91,632					
10										

Note: the Boolean FALSE (fourth argument) tells the VLOOKUP function to return an exact match. If the VLOOKUP function cannot find the value 53 in the first column, it will return a #N/A error

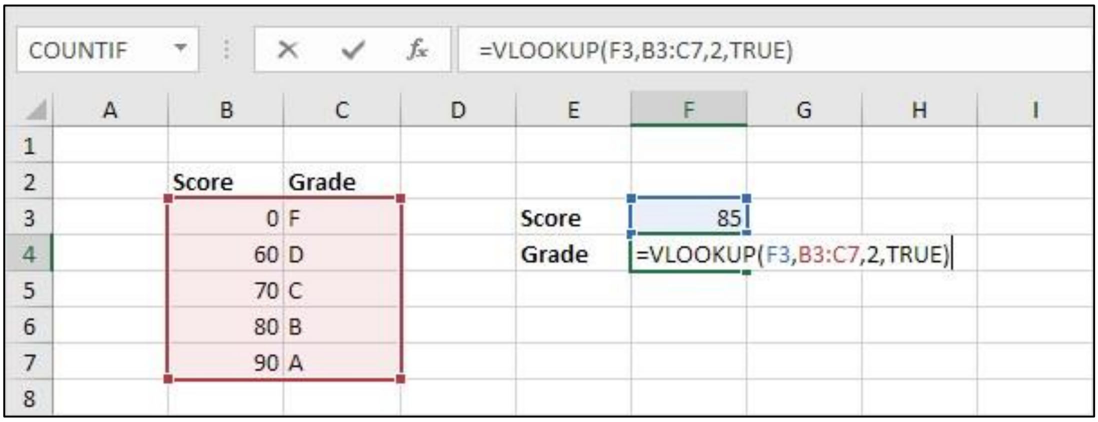
(c) Here's another example. Instead of returning the salary, the VLOOKUP function below returns the last name (third argument is set to 3) of ID 79.



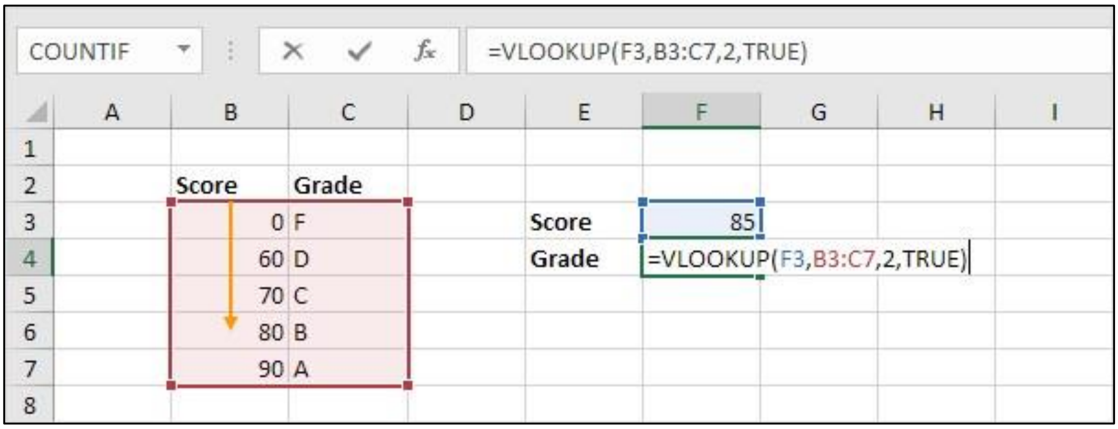
H3 =VLOOKUP(H2,B3:E9,3,FALSE)

	A	B	C	D	E	F	G	H	I	J
1		1	2	3	4					
2		ID	First Name	Last Name	Salary		ID	79		
3		72	Emily	Smith	$64,901		Last Name	Lopez		
4		66	James	Anderson	$70,855					
5		14	Mia	Clark	$188,657					
6		30	John	Lewis	$97,566					
7		53	Jessica	Walker	$58,339					
8		56	Mark	Reed	$125,180					
9		79	Richard	Lopez	$91,632					
10										

(d) **Approximate Match.** Let's take a look at an example of the VLOOKUP function in approximate match mode (fourth argument set to TRUE). The VLOOKUP function below looks up the value 85 (first argument) in the leftmost column of the red table (second argument). There's just one problem. There's no value 85 in the first column.



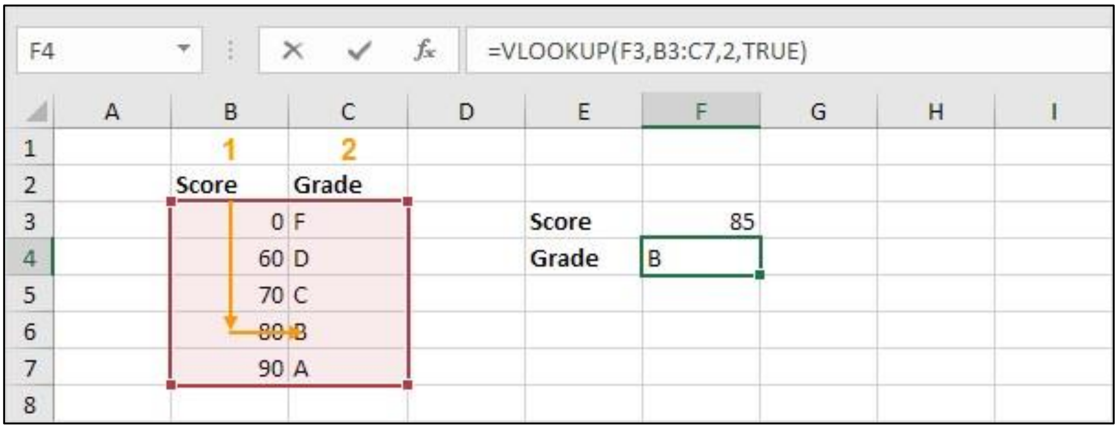
Fortunately, the Boolean TRUE (fourth argument) tells the VLOOKUP function to return an approximate match. If the VLOOKUP function cannot find the value



85 in the first column, it will return the largest value smaller than 85. In this example, this will be the value 80.

(e)

The value 2 (third argument) tells the VLOOKUP function to return the value in the same row from the second column of the red table.



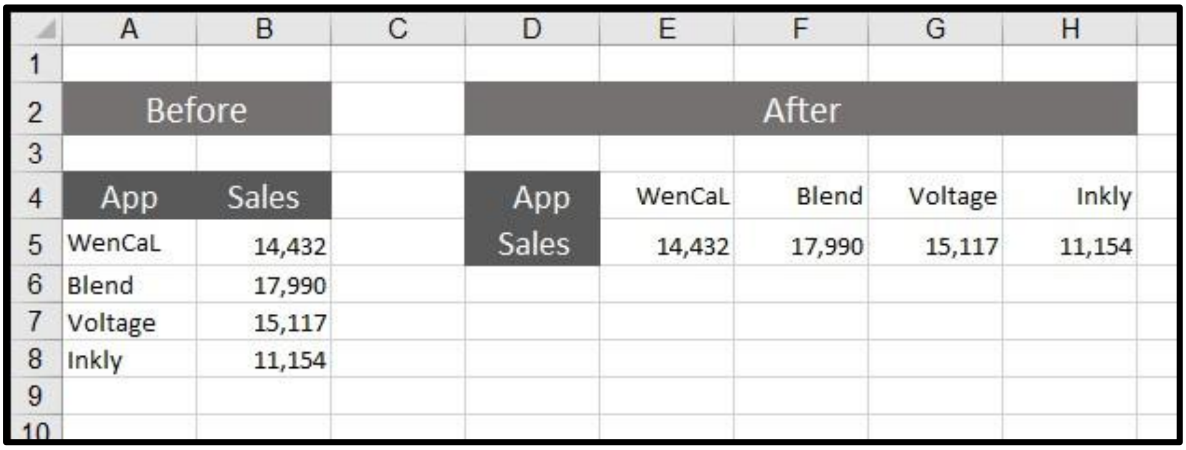
(e) Note: always sort the leftmost column of the red table in ascending order if you use the VLOOKUP function in approximate match mode (fourth argument set to TRUE).

(f) Vlookup looks right.

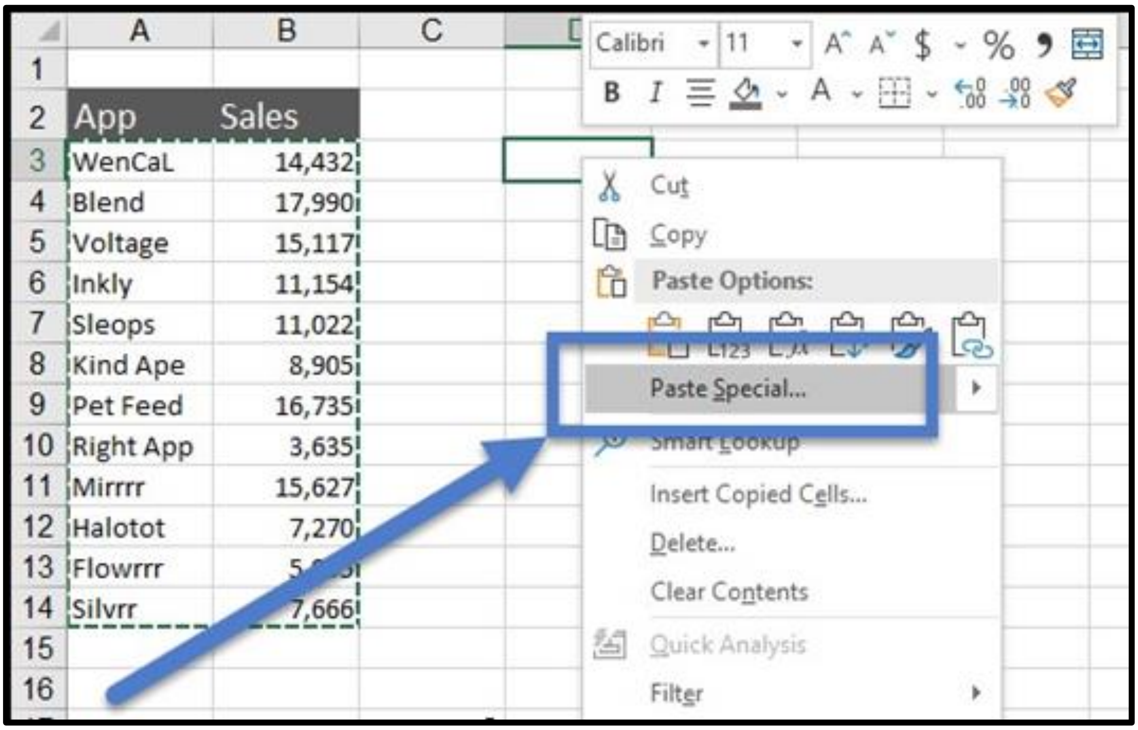
(g) Vlookup is Case-insensitive.

(h) The secret to VLOOKUP is to organize your data so that the value you look up is to the left of the return value you want to find.

22. **Transpose**. The Excel TRANSPOSE function "flips" the orientation of a given range or array. TRANSPOSE converts a vertical range to a horizontal range, or a horizontal range to a vertical range. You must enter the TRANSPOSE function as an array formula. Transposing data is where the data in the rows are turned into columns, and the data in the columns is turned into rows.



23. Transpose can be executed by two methods; Static and Dynamic.

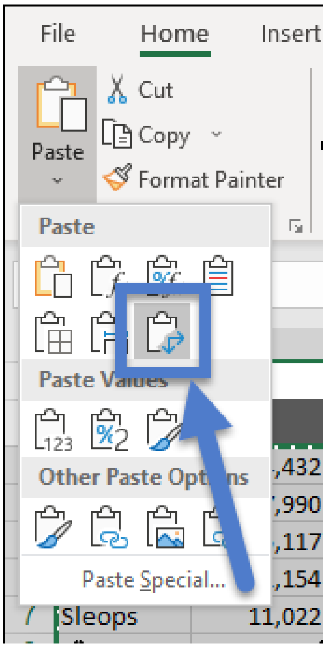


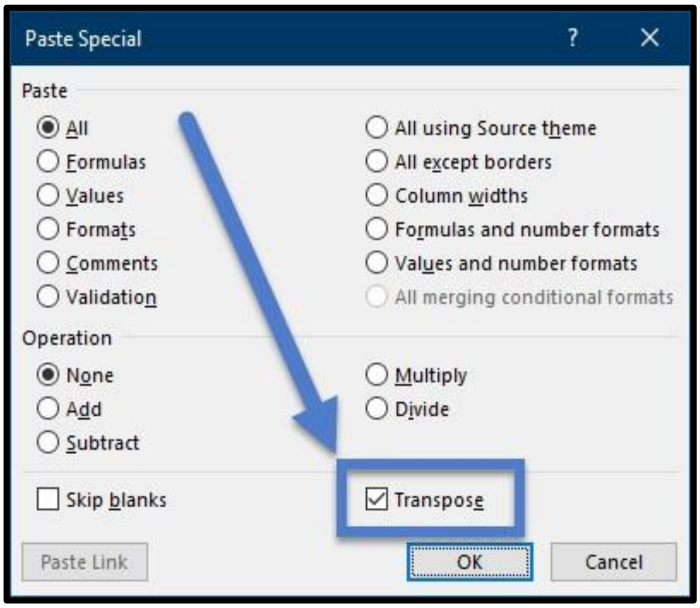
(a) **Static**. It implies that the result is a static solution (meaning the transposed data will not update if the original data changes).

(i) Select the data (A3:B14) and press the Ctrl + C or the copy button.

(ii) Select the destination cell that will serve as upper left corner of the transposed data (D3) and right mouse click to paste special.

(iii) Check the box labelled 'Transpose' & click OK.



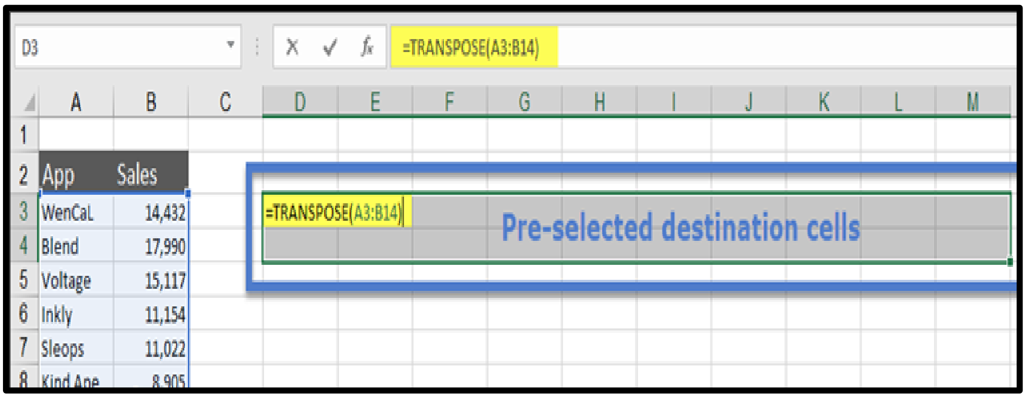


NOTE: An alternative to this is to select the data and click Copy, click the destination cell, then click the lower part of the Paste button and click the Transpose. T he downside to this method is that it is not dynamic. If data is changed in the original "vertical" data set, the corresponding entry in the "horizontal" data set will not reflect the change.

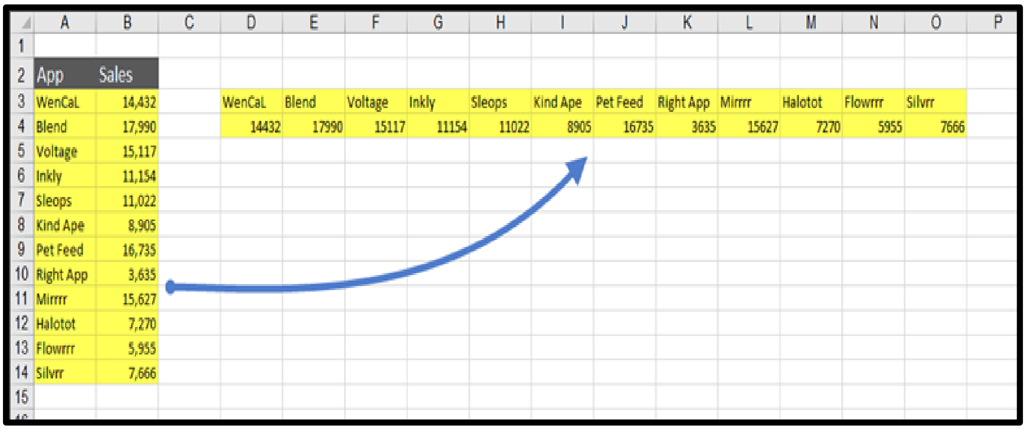
(b) **Dynamic**. TRANSPOSE is an array formula and because of this, we need to select beforehand a range of cells that will serve as a landing zone for all the possible answers.

(i) Highlight a range of cells that can support the returned values (i.e. **D3:O4**).

(ii) Enter the formula =TRANSPOSE(A3:B14) (***do NOT hit ENTER!!!***):



(iii) Press **CTRL-Shift-Enter**



(iv) Because this is a formula, when data changes in the original “vertical” list, we see the same change to data in the “horizontal” list.

24. __Index__. This function returns the value of an element in a table or an array, selected by the row and column number that you specify. Syntax is **INDEX(array, row_num, [column_num])**

(a) **__Array__.** It is a range of cells, named range, or table.

(b) **__Row_num__.** It is the row number in the array from which to return a value. If row_num is omitted, column_num is required.

(c) **__Column_num__.** It is the column number from which to return a value. If column_num is omitted, row_num is required.

25. For example, the formula **=INDEX($B$3:$F$7,1,2)** in cell B10, returns the value at the intersection of the 1st row and 2nd column in range B3:F5, which is 100.

IF =INDEX($B$3:$F$7,B9,C9)

	A	B	C	D	E	F	G
1		TO LOC					
2	FROM LOC	1	2	3	4	5	
3	1	-	100	85	140	120	
4	2	88	-	42	125	165	
5	3	70	50	-	170	136	
6	4	165	150	190	-	98	
7	5	105	180	160	90	-	
8							
9		1	2	3	4	5	1
10		100	42	170	98	105	515
11		=INDEX($B$3:$F$7,B9,C9)					

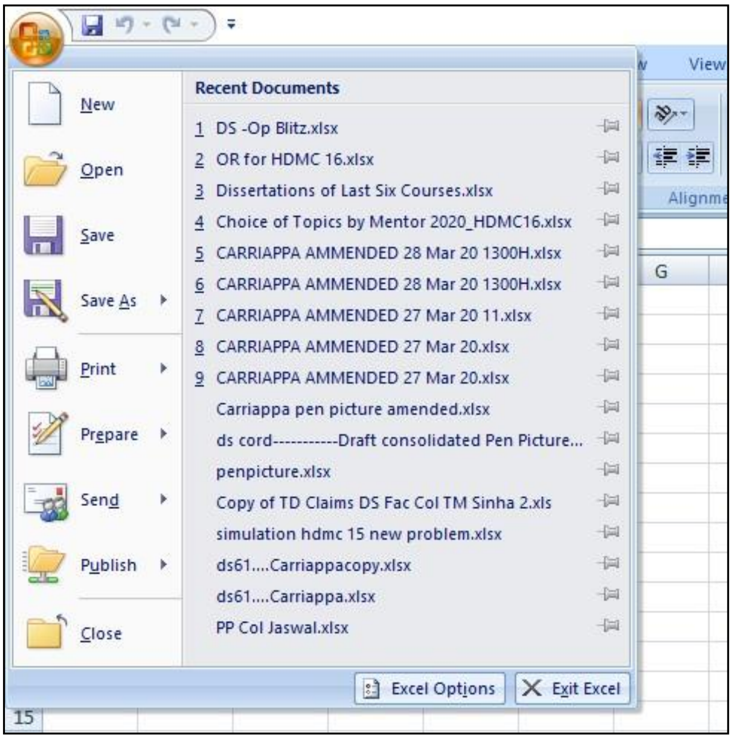
Instead of entering the row and column numbers in the formula, you can supply the cell references to get a more universal formula: **=INDEX($B$3:$F$7,B9,C9)** So, this INDEX formula returns the value at the intersection of the row and col number specified by cell B9 and C9.

(a) If both the row_num and column_num arguments are used, INDEX returns the value in the cell at the intersection of row_num and column_num.

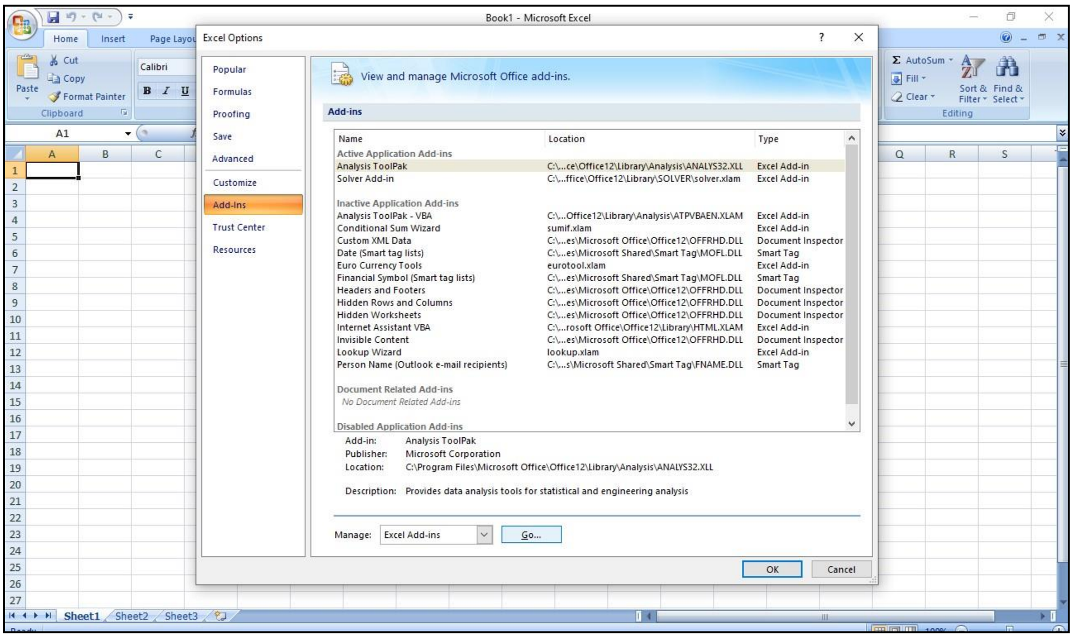
(b) Row_num and column_num must point to a cell within array; otherwise, INDEX returns a #REF! error.

(c) If you set row_num or column_num to 0 (zero), INDEX returns the array of values for the entire column or row, respectively. To use values returned as an array, enter the INDEX function as an array formula.

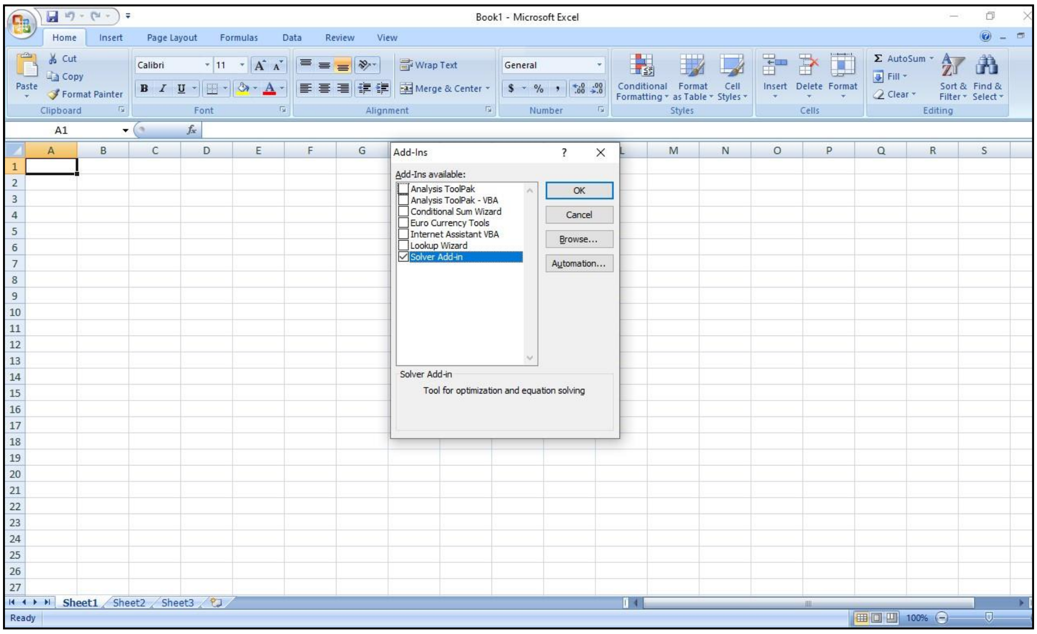
26. **Excel Solver**. Excel solver is an 'ADD IN' in the MS Excel that is used extensively to solve OR problems and needs to be initialised before it can be used. The steps for initializing the solver are as below:- (a) Click on Office button and go to Excel Options.



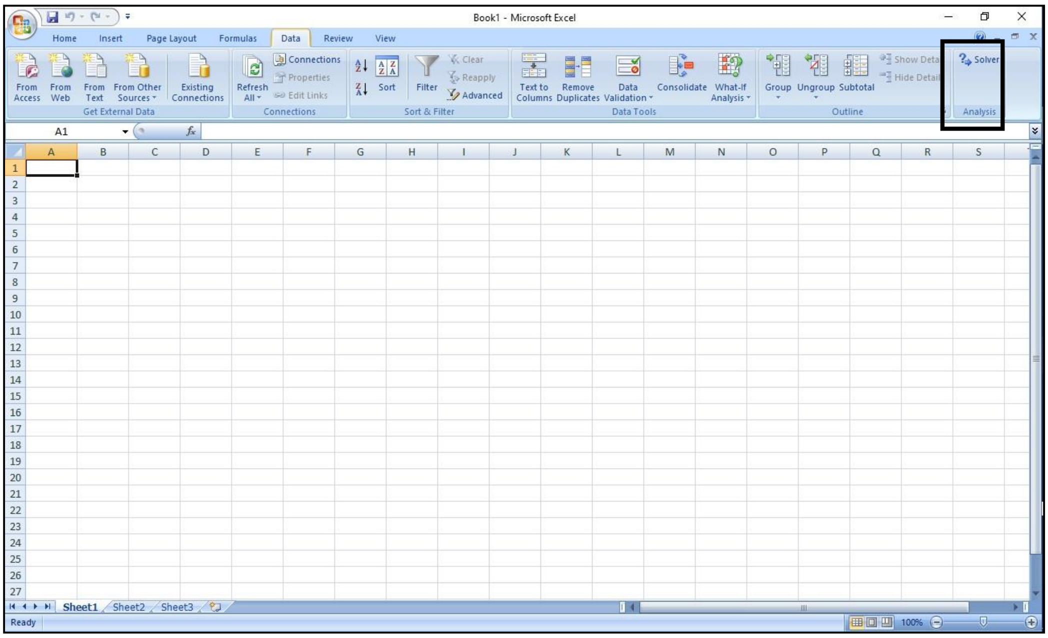
(b) Click on Add-Ins and select Excel Add-Ins under drop down menu of Manage in the bottom of dialogue box. Click on Go.



(c) It will open a Add-Ins window with seven options. Select Solver Add In and click OK.



(d) This will add Solver under the Data Tab of excel ribbon.



27. **<u>Exploring Excel Solver</u>**. All OR problems invariably have to be reduced to a mathematical model which can be solved for optimization using MS Excel Solver or other softwares. A problem whose mathematical model is as below, can be solved using Solver following steps as shown in succeeding paragraphs:-

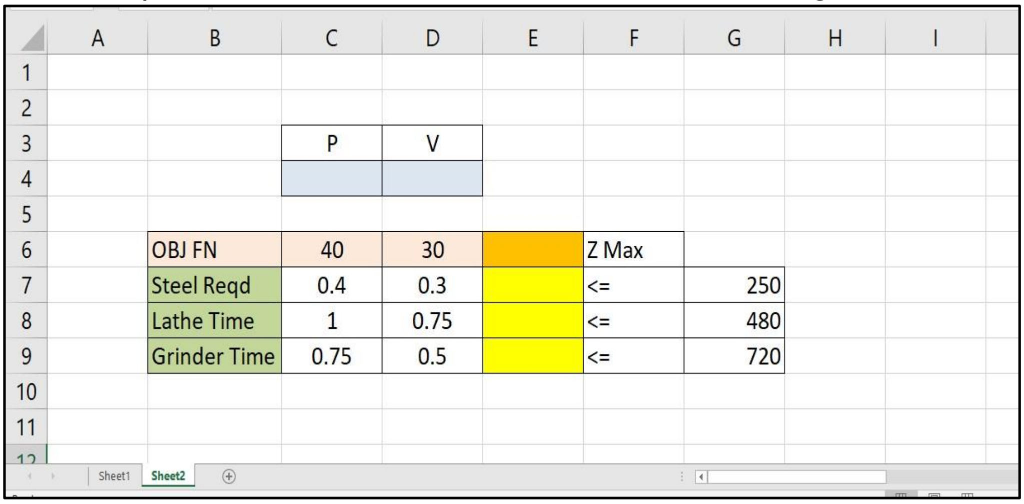
Zmax = 40 P + 30 V

Subject to:

Steel Required : 0.4 P + 0.3 V ≤ 250

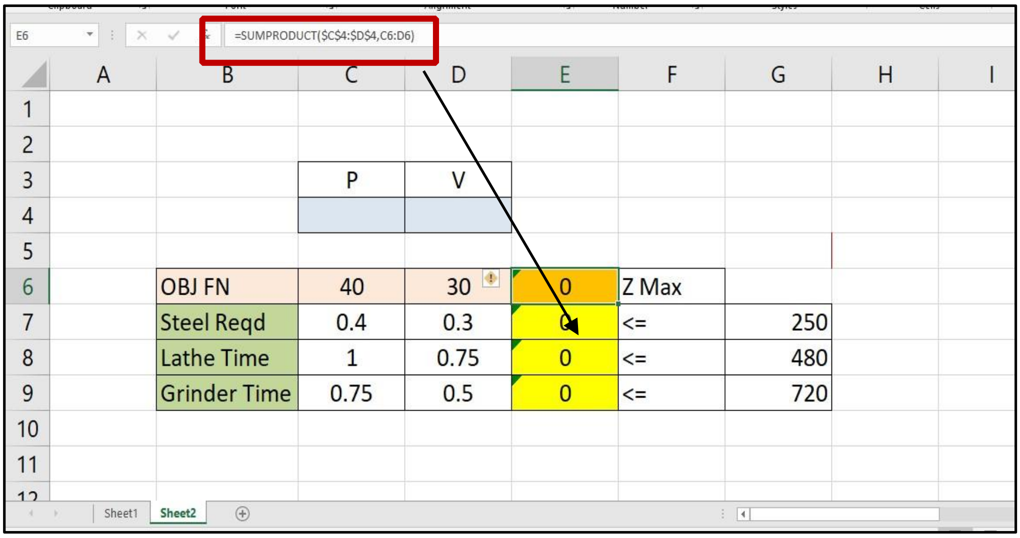
Lathe Time	:	1P + 0.75 V	≤ 480
Grinder Time	:	0.75 P + 0.5 V	≤ 720
Non Negativity	:	P, V	≥ 0

28. Feed the problem in a normal Excel sheet as shown in Fig below:-



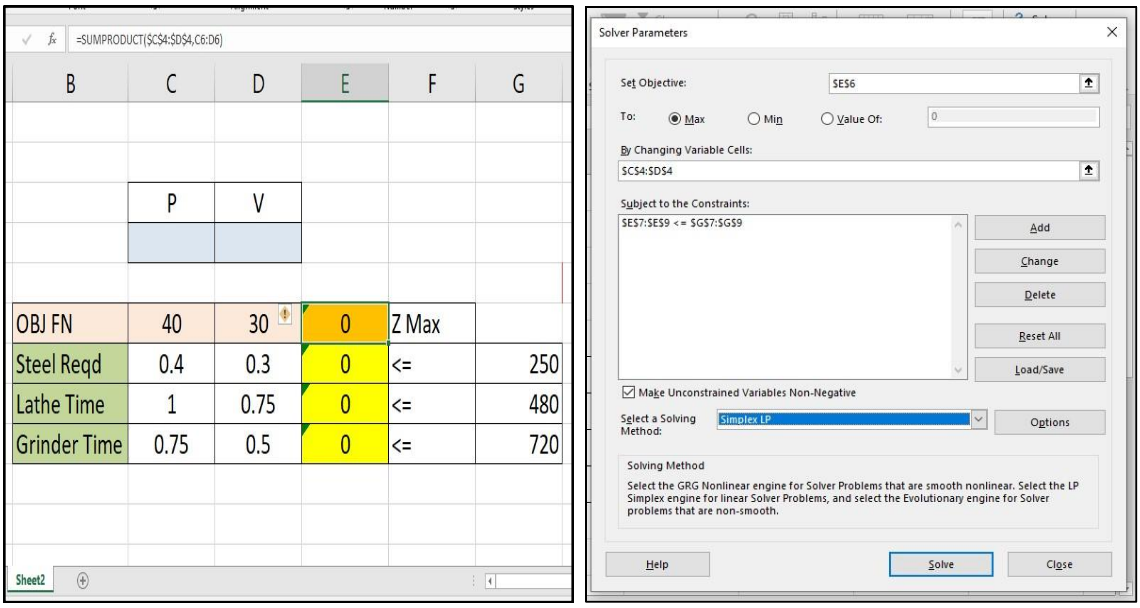
	A	B	C	D	E	F	G	H	I
1									
2									
3			P	V					
4									
5									
6		OBJ FN	40	30		Z Max			
7		Steel Reqd	0.4	0.3		<=	250		
8		Lathe Time	1	0.75		<=	480		
9		Grinder Time	0.75	0.5		<=	720		
10									
11									

29. While being in E6, Select SUMPRODUCT. Select C4:D4 in the first array or type in the formula bar and press F4. This step ensures that C4:D4 cells remain constant for all array multiplication. In the second array or after comma, select C6:D6 and enter. You will get Zero as the Sum-Product in the cell (E6). Now drag the right bottom corner portion of this cell (E6) down upto cell E9 to get SUMPRODUCT for all constraints equations as shown in Fig below:-



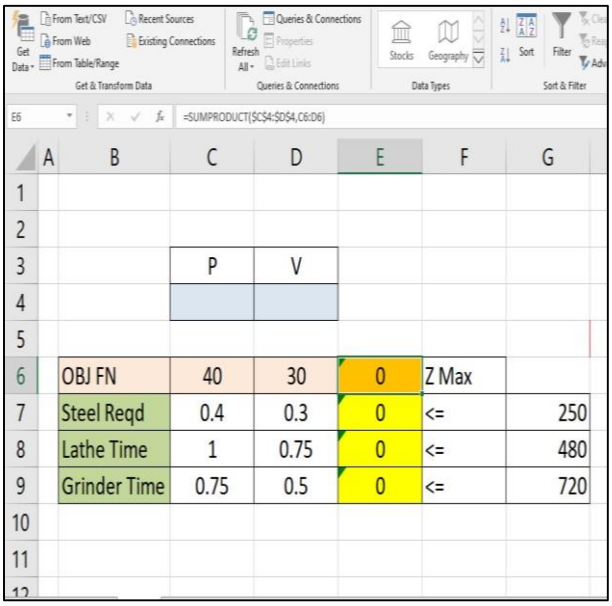
	A	B	C	D	E	F	G	H	I
1									
2									
3			P	V					
4									
5									
6		OBJ FN	40	30	0	Z Max			
7		Steel Reqd	0.4	0.3	0	<=	250		
8		Lathe Time	1	0.75	0	<=	480		
9		Grinder Time	0.75	0.5	0	<=	720		
10									
11									

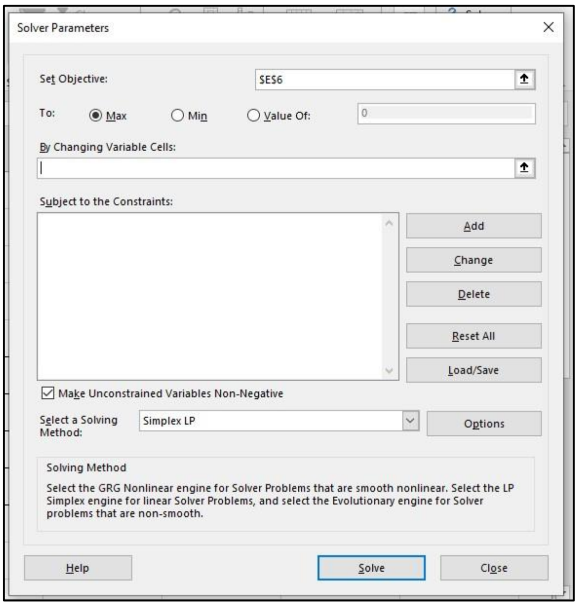
30. While being in cell E6 Click on Data in Menu bar and select Solver as shown in Fig below.



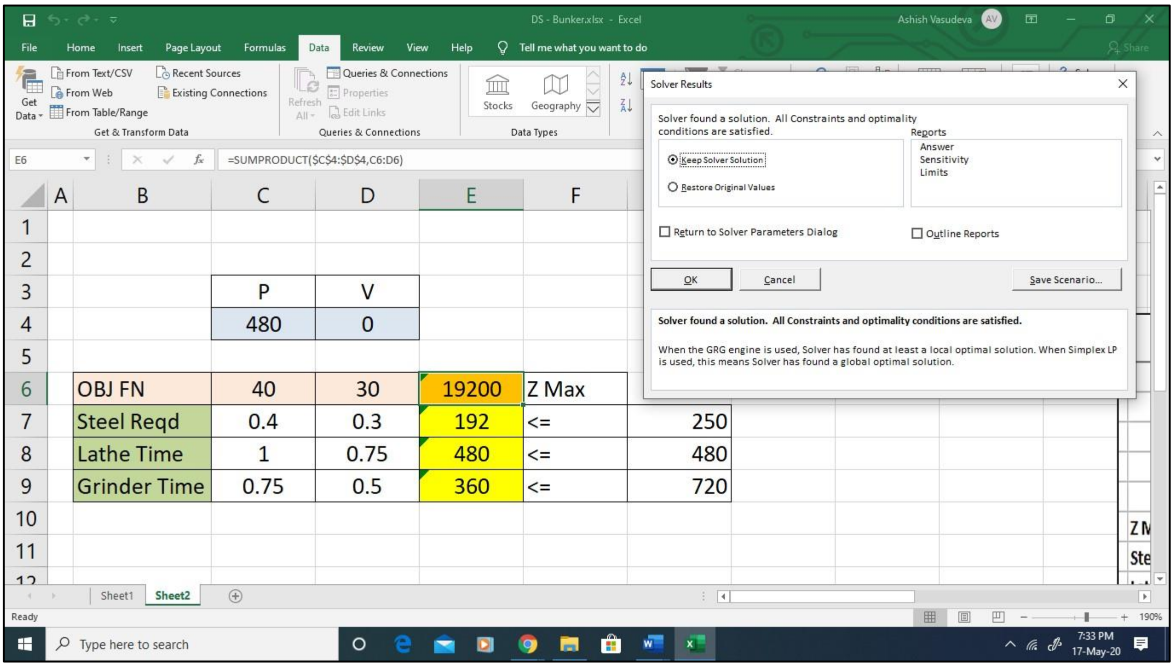
31. In Solver dialogue box enter the following: -

'Set Objective'	Select cell E6 (where you want Objective function value to come)
'To'	Click on Max in this case.
'By Changing Variable'	Select cells C4:D4 (where you want Decision Cells' Variable values to come).
'Subject to the Constraints'	First click on 'Add' button.
	Select cells E7:E9 under 'Cell reference'; Select <= sign ;
	Under 'Constraints' select G7:G9.
	Then click 'OK' button (if all constraints have been added).
	Check the Box : Make Unconstrained Variables Non Negative as shown in Fig below:-





32. Click on 'Solve' button. You may also select 'Answer', 'Sensitivity' and 'Limits' in Solver Results window if required. Select Keep Solver Solution. 31. Press OK, you will get the following screen as shown in Fig below.



33. The Excel Solver has given a solution to fabricate 480 spares of type P and Nil of type V. This combination would accrue a Max profit of Rs 19200. No other combination of spares of type P and V would give you more profit.

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